The Product and Quotient Rules

Math 130 - Essentials of Calculus

14 October 2019

EXPANDING OUR ABILITIES

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THEOREM (THE PRODUCT RULE)

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x).$$

EXAMPLES

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Differentiate the following functions

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2. $g(x) = \sqrt{x} e^x$
3. $R(t) = (t + e^t)(3 - \sqrt{t})$
4. $k(x) = \dfrac{e^x}{x^2}$

MORE EXAMPLES

EXAMPLE

If $f(x) = \sqrt{x}g(x)$, with $g(4) = 2$ and $g'(4) = 3$. Find $f'(4)$.

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EXAMPLE

Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. If $A(x) = f(x)g(x)$, find $A'(5)$.

THE QUOTIENT RULE

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THEOREM (THE QUOTIENT RULE)

If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}.$$

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Differentiate the given function:

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2. $g(x) = \dfrac{3x-1}{2x+1}$

3. $f(x) = \dfrac{t^3 + t}{t^4 - 2}$

4. $L(t) = \dfrac{t^2}{3t^2 - 2t + 1}$

MORE EXAMPLES

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Compute the second derivative of the following functions:

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Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. If $C(x) = \dfrac{g(x)}{f(x)}$, find $C'(5)$.